

The Internal Ballistics of an Air Gun

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The internal ballistics of a firearm or artillery piece considers the pellet, bullet, or shell motion while it is still inside the barrel. In general, deriving the muzzle speed of a gunpowder firearm from first principles is difficult because powder combustion is fast and it very rapidly raises the temperature of gas (generated by gunpowder *deflagration*, or burning), which greatly complicates the analysis. A simple case is provided by air guns, for which we can make reasonable approximations that permit a derivation of muzzle speed. It is perhaps surprising that muzzle speed depends upon barrel length (artillerymen debated this dependence for centuries, until it was established experimentally and, later, theoretically¹). Here we see that a simple physical analysis, accessible to high school or freshmen undergraduate physics students, not only derives realistic muzzle speed but also shows how it depends upon barrel length.

The analysis is simpler for air guns because we can plausibly make the assumption that temperature is constant during the gas expansion phases, so that Boyle's law applies instead of a more complicated temperature-dependent gas law.² The trigger releases compressed gas; pressure from this gas projects the pellet down the gun barrel. We can neglect the effects of aerodynamic drag within the barrel, but we must account for contact friction between pellet and barrel. Here the friction force is assumed to be constant (independent of pellet speed). This may be unrealistic for skirted (*diabolo*) lead pellets: these are designed so that gas pressure expands the skirt to seal the bore, so that power is not wasted as gas escapes through a gap between pellet and barrel.³ It may be the case that the normal force between pellet and barrel depends upon gas pressure. Three other assumptions: the seal is perfect, the energy consumed in spinning the pellet as it travels down the rifled barrel is negligible, and the energy consumed in accelerating the gas is negligible. These last assumptions are all reasonable,⁴ but assuming constant friction is questionable and assuming isothermal expansion especially so. Thus, we can anticipate that the prediction of our simple model will be approximate. The simplifications that our assumptions permit mean that we can derive an analytic expression for muzzle speed from our simple model.

Muzzle speed and barrel length

There are several variants of air gun that differ in the means by which air is compressed. The action of spring piston airguns is complex to analyze, and so we will not consider them further.⁵ For pneumatic guns, air is compressed via a cocking lever; pre-charged pneumatic (PCP) guns dispense with the lever and obtain compressed air directly from an ex-

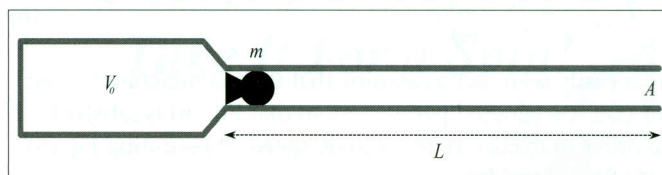


Fig. 1. Illustration of an air gun. Barrel length is L and cross-sectional area is $A = \pi(c/2)^2$, where c is caliber. The gas reservoir has volume V_0 before the trigger is pulled. Pellet mass is m .

ternal cylinder such as a diver's tank; carbon dioxide guns employ a small powerlet canister containing liquid CO_2 . These latter three types of gun all charge a reservoir with gas under pressure (ranging from 70-200 atm). This is the system we analyze here. In Fig. 1 you can see the reservoir of compressed gas (volume V_0 and pressure P_0). When the trigger is pulled, this gas expands into the barrel (of cross-sectional area A and length L), pushing out the pellet (mass m). We are assuming that gas temperature does not change appreciably during this expansion phase, so that Boyle's law applies:

$$P(t)V(t) = P_0V_0, \quad (1)$$

where $P(t)$ and $V(t)$ are the gas pressure and volume at a time t after the trigger is pulled, and where $P(0) = P_0$, etc. From Fig. (1) we see that

$$V(t) = V_0 + Ax(t), \quad (2)$$

where $x(t)$ is the position of the pellet in the barrel ($0 \leq x \leq L$). In Fig. 1 the pellet is in its initial position, $x(0) = 0$. The force acting upon the pellet is thus

$$\begin{aligned} F &= AP(t) = \frac{AP_0V_0}{V_0 + Ax(t)} - f \\ &= m\ddot{x}(t) \end{aligned} \quad (3)$$

In Eq. (3) we adopt Newton's dot notation for time derivative; f is the constant friction force between barrel and pellet. This equation is separable, because $\ddot{x} = \dot{x} d\dot{x}/dx$, and easy to integrate:

$$\frac{1}{2}mv^2(x) = P_0V_0 \ln \left(1 + \frac{Ax}{V_0} \right) - fx, \quad (4)$$

where $v(x) = \dot{x}$. Note that the first term on the right is the energy (W) released by the gas as it expands from its initial volume to position x in the barrel, and the second term is the energy dissipated by friction between pellet and barrel. The term on the left is pellet (kinetic) energy. For our simplified analysis, there is no energy dissipated in heating the gas.

Pellet muzzle speed is

$$v(L) = \sqrt{\frac{2}{m} \left[P_0 V_0 \ln \left(1 + \frac{AL}{V_0} \right) - fL \right]} \quad (5)$$

The muzzle speed of Eq. (5) peaks for a barrel length given by

$$L_{\max} = \frac{P_0 V_0}{f} - \frac{V_0}{A}, \quad (6)$$

as is easily seen. Let us assume that the manufacturer of our air gun has adopted parameters so that Eq. (6) is satisfied, in order to maximize the muzzle speed. Substituting Eq. (6) into Eq. (5) yields

$$v(f) = \sqrt{\frac{2}{m} P_0 V_0 \left(\ln \left(\frac{P_0 A}{f} \right) - 1 - \frac{f}{P_0 A} \right)}. \quad (7)$$

Equations (6) and (7) are plotted in Figs. 2(a) and (b). Figure 2(c) shows what happens when we eliminate f and plot $v(L_{\max})$. Thus, we obtain muzzle speed as a function of barrel length for this type of gun. We see that muzzle speed increases as barrel length increases, and that the rate of increase slackens somewhat for longer barrels; this type of behavior is also observed in gunpowder weapons, except that the reduction in acceleration is more marked.

For the graphs plotted in Fig. 2, we have adopted the following parameter values: $P_0 = 200 \text{ atm}$ ($2 \times 10^7 \text{ Nm}^{-2}$); $V_0 = 16 \text{ cc}$ ($1.6 \times 10^{-5} \text{ m}^3$), typical for a CO_2 powerlet; $A = 1.6 \times 10^{-5} \text{ m}^2$ for a .177 caliber gun, and $A = 2.5 \times 10^{-5} \text{ m}^2$ for a .22 caliber gun; $m = 8 \text{ grains}$ ($5.2 \times 10^{-4} \text{ kg}$) for a .177 pellet, and $m = 15 \text{ grains}$ ($1.0 \times 10^{-3} \text{ kg}$) for a .22 pellet. Note also in Fig. 2 that we have restricted the lengths of barrels to a minimum of 15 cm (air pistol) and a maximum of 50 cm (air rifle). All these figures are representative and they result in realistic muzzle speeds.

The efficiency ε of our air gun is the ratio of pellet energy to gas energy at the muzzle, which is easily seen from the foregoing to be

$$\varepsilon = \frac{\frac{1}{2}mv^2}{W} = 1 - \frac{z}{(1+z)\ln(1+z)}, \quad z \equiv \frac{AL_{\max}}{V_0}. \quad (8)$$

For our parameter values we find that $\varepsilon = 15\%$ for the .177 gun and $\varepsilon = 24\%$ for the .22 gun: most of the energy is expended overcoming friction. Why so inefficient if the muzzle speed has been optimized? Longer barrels mean more energy diverted to friction, whereas shorter barrels mean more gas pressure is wasted: for our choice of parameters we find [from Eqs. (1)-(3)] that gas pressure at the muzzle $P(L)$ is 71% of the initial gas pressure P_0 for a .177 gun (56% for a .22 gun). Once the pellet leaves the barrel, the remaining gas pressure is dissipated in the air. So, the optimum barrel length is a trade-off between friction and lost gas pressure.

Note from Figs. 2(a) and (b) that, for unrealistically long barrels, our simple model predicts unrealistically high muzzle speed. (We expect the upper limit of muzzle speed for any air gun projectile to be the speed of sound in air, as this is the

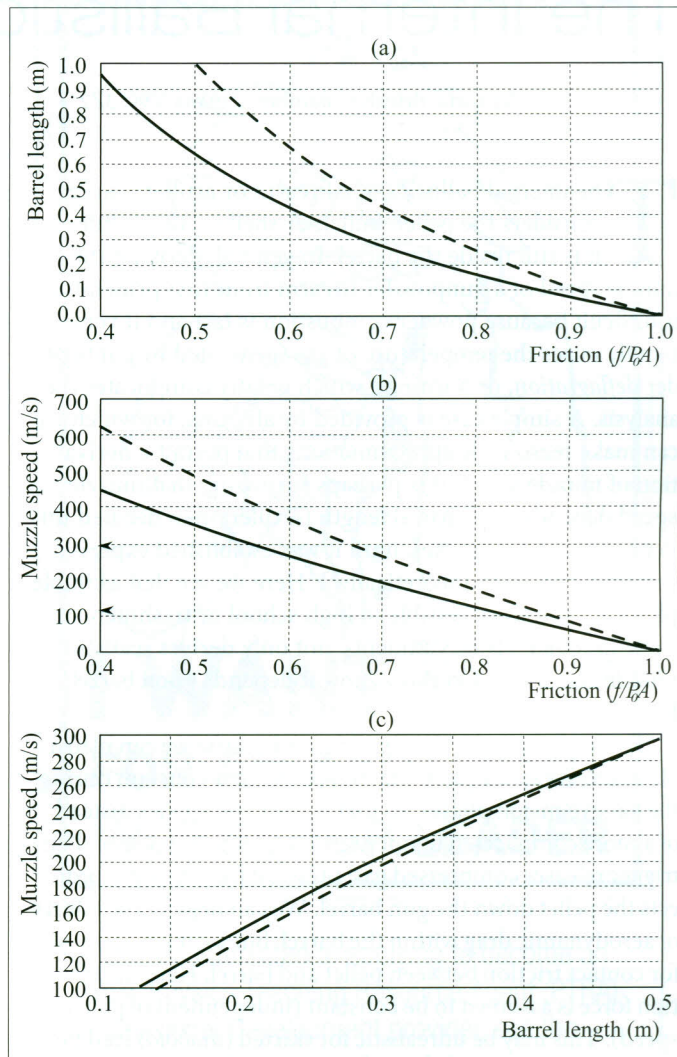


Fig. 2. (a) Air gun barrel length vs dimensionless friction for optimized muzzle speed. Plots shown are for a .22 caliber gun (solid line) and for a .177 caliber gun (dashed line). (b) Optimized muzzle speed vs dimensionless friction. (c) Muzzle speed vs barrel length for a small section of (a) and (b) restricted to practical values of barrel length (approximately 0.15 m to 0.5 m).

speed of the pressure waves.) In this region our assumption of isothermal expansion breaks down. If expansion causes a gas to change temperature, we may ignore the change for small expansions (short barrel lengths) but not for more significant expansions. Thus, our simple model breaks down for long barrel lengths.

Summary

This brief exercise shows that air gun muzzle speed increases with barrel length and predicts realistic values for muzzle speed given realistic input parameters. Unrealistic values arise for situations where the isothermal expansion assumption breaks down. (The interested student may wish to repeat our analysis assuming adiabatic expansion of an ideal gas instead of isothermal expansion. This exercise leads to a different expression for muzzle speed as a function of barrel

length.) The calculation involves elementary but important physical concepts and techniques, including mechanical and thermodynamic energy, contact friction, force, elementary differential equations, and optimization. Applying these techniques to a practical problem illustrates their applicability and engages the interest of students perhaps more than would a dry textbook example.

Acknowledgment

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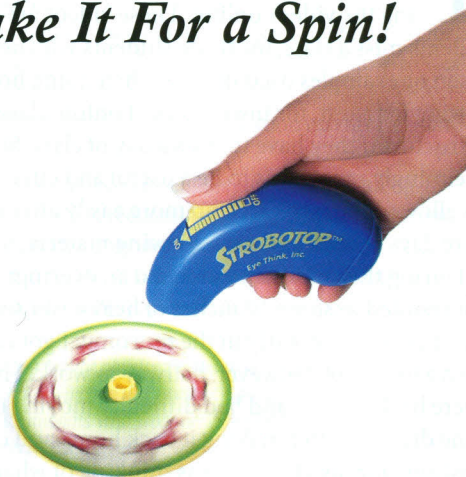
References

1. See, for example, Bert S. Hall, *Weapons and Warfare in Renaissance Europe* (Johns Hopkins University Press, Baltimore, 1992); Captain J. G. Benton, *Ordnance and Gunnery; Compiled for the use of the Cadets of the United States Military Academy* (D. van Nostrand, New York, 1862).
2. Thus our calculation is based upon isothermal gas expansion. This contrasts with the analysis of Mungan, who assumed adiabatic expansion. See Carl E. Mungan, "Internal ballistics of a pneumatic potato cannon," *Eur. J. Phys.* **30**, 453–457 (2009).
3. The difference between projectile diameter and barrel diameter for an old-fashioned musket is called *windage*. Windage was unavoidable for these muzzleloaders, even though it wasted gas pressure, because a tight-fitting ball could not be rammed down the barrel. An air rifle pellet expands to fill the windage gap, much as did a "Minié ball"—the bullet fired from a Civil War-era rifled musket. See, for example, Barton C. Hacker, *American Military Technology* (Johns Hopkins University Press, Baltimore, 2006), p. 21.
4. From the twist of the rifling and the muzzle speed of a pellet, it is straightforward to determine the projectile angular speed—typically this is two orders of magnitude below the pellet translational energy. *Powerlets* (cylinders containing 12 g of liquid CO₂ that are a common source of gas for low-caliber air guns) last for about 40 shots before the pressure drops unacceptably, and so they expend *less than* 5 grains of gas per shot—a third of the weight of a .22 pellet.
5. Spring-loaded air guns are analyzed in an online article (home2.fvcc.edu/~dhicketh/Math222/spring07/projects/StephenCompton/SpringAirModel.pdf) in which MATLAB is employed to solve the equations.

Mark Denny obtained a PhD in theoretical physics from Edinburgh University, Scotland, and spent 20 years working as a radar systems engineer. He now writes popular science books; his first book (*Ingenium: Five Machines that Changed the World*) includes a chapter about siege engines, and is published by Johns Hopkins University Press.
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